# Premio “Vincenza Celluprica”

## IX edizione-2016

PhD thesis in “Fundamental and Applied Physics”

## Extreme Regimes in Quantum Gravity


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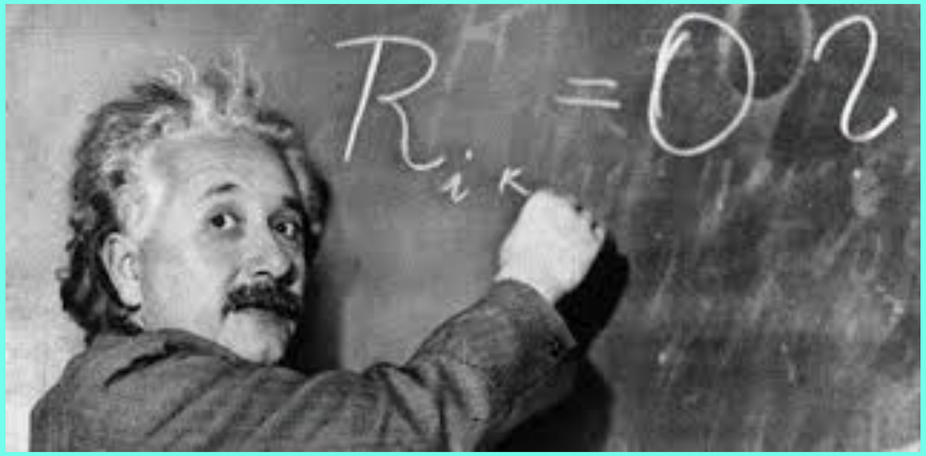
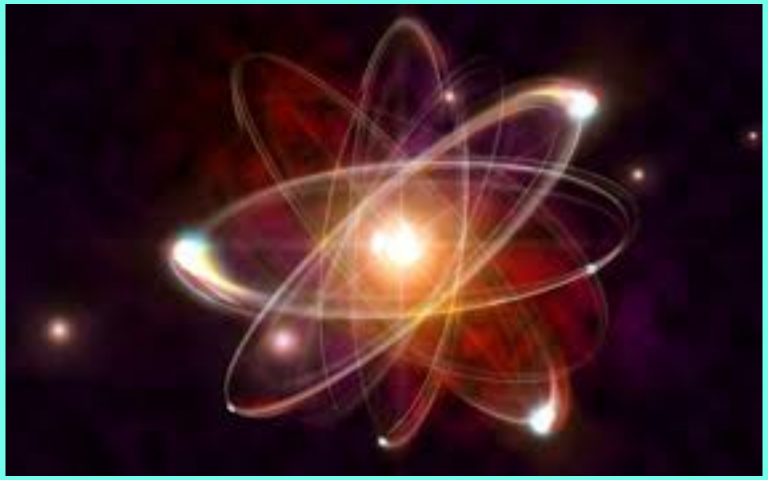
# The title of the thesis: why "Quantum Gravity"?

- General Relativity: one of the most elegant and exciting theory of theoretical physics describing the features of the gravitational fields.



$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

**+**



- Quantum field theory: a synthesis of quantum mechanics and <u>special relativity</u> describing physical phenomena at microscopic level.

**= ?**

# The title of the thesis: why "Extreme"?

Thesis divided into two parts:

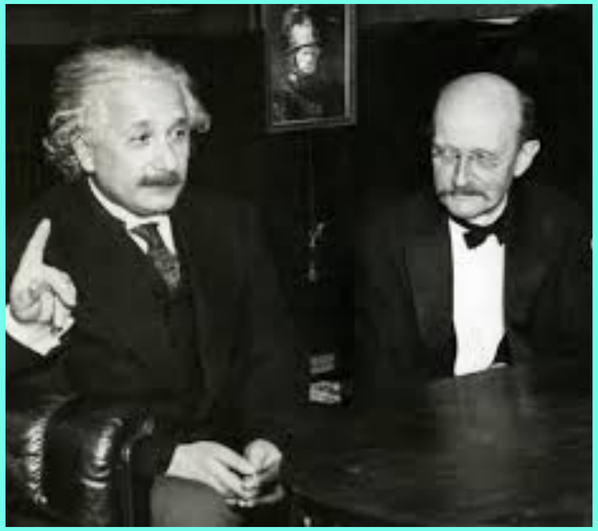
- First part: low-energy limit of quantum gravity:
    - Quantum gravity as an effective field theory
    - The restricted three-body problem:
        1. Quantum effects on Lagrangian points

- Second part: high-energy limit of quantum gravity:
    - The Boost of a metric:
        1. Gravitational shock-wave
        2. The Boosted Schwarzschild-de Sitter metric
        3. The "boosted horizon" and the singularity 3-sphere

# Effective field theory

In the absence of a viable theory of quantum gravity, is it possible to describe some effects involving the gravitational field at quantum level?

The answer is yes!



Effective field theory

- An Effective Field Theory (EFT) is a type of approximation to a quantum field theory.

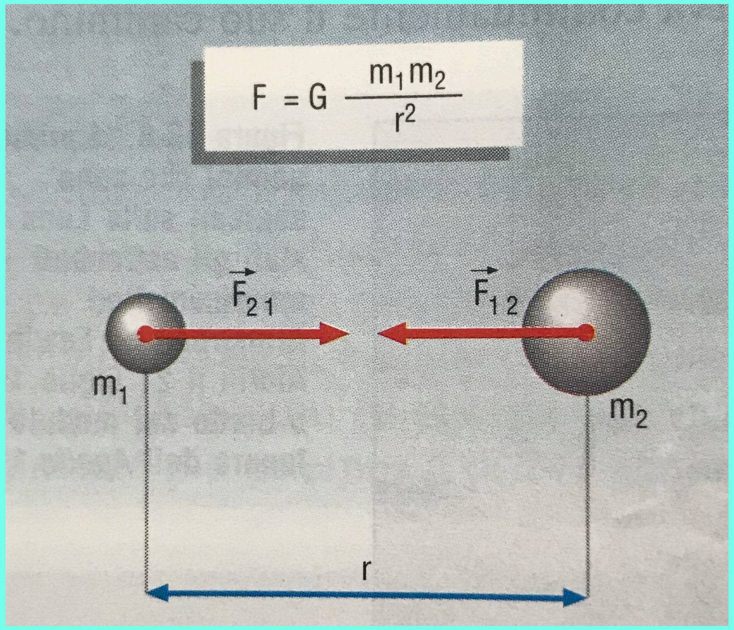
- An EFT includes the appropriate degrees of freedom to describe physical phenomena occurring at a chosen energy scale, while ignoring substructure and degrees of freedom at higher energies.

# Quantum corrected potential

Within context of effective field theory, it is possible to derive the leading quantum corrections to the Newtonian potential:

$$V_N(r) = -\frac{G m_A m_B}{r}$$

Newtonian potential




$$V_E(r) = -\frac{G m_A m_B}{r}\left[1 + \left(\frac{k_1}{r} + \frac{k_2}{r^2}\right) + \mathrm{O}(G^2)\right]$$

Quantum corrected potential

| $\kappa_i$ | one-particle reducible | scattering | bound-states |
|---|---|---|---|
| $\kappa_1$ | $-1$ | $3$ | $-\frac{1}{2}$ |
| $\kappa_2$ | $-\frac{167}{30\pi}$ | $\frac{41}{10\pi}$ | $\frac{41}{10\pi}$ |

$$k_1 \equiv \kappa_1 \frac{G(m_A + m_B)}{c^2} = \kappa_1\left(R_A + R_B\right),$$

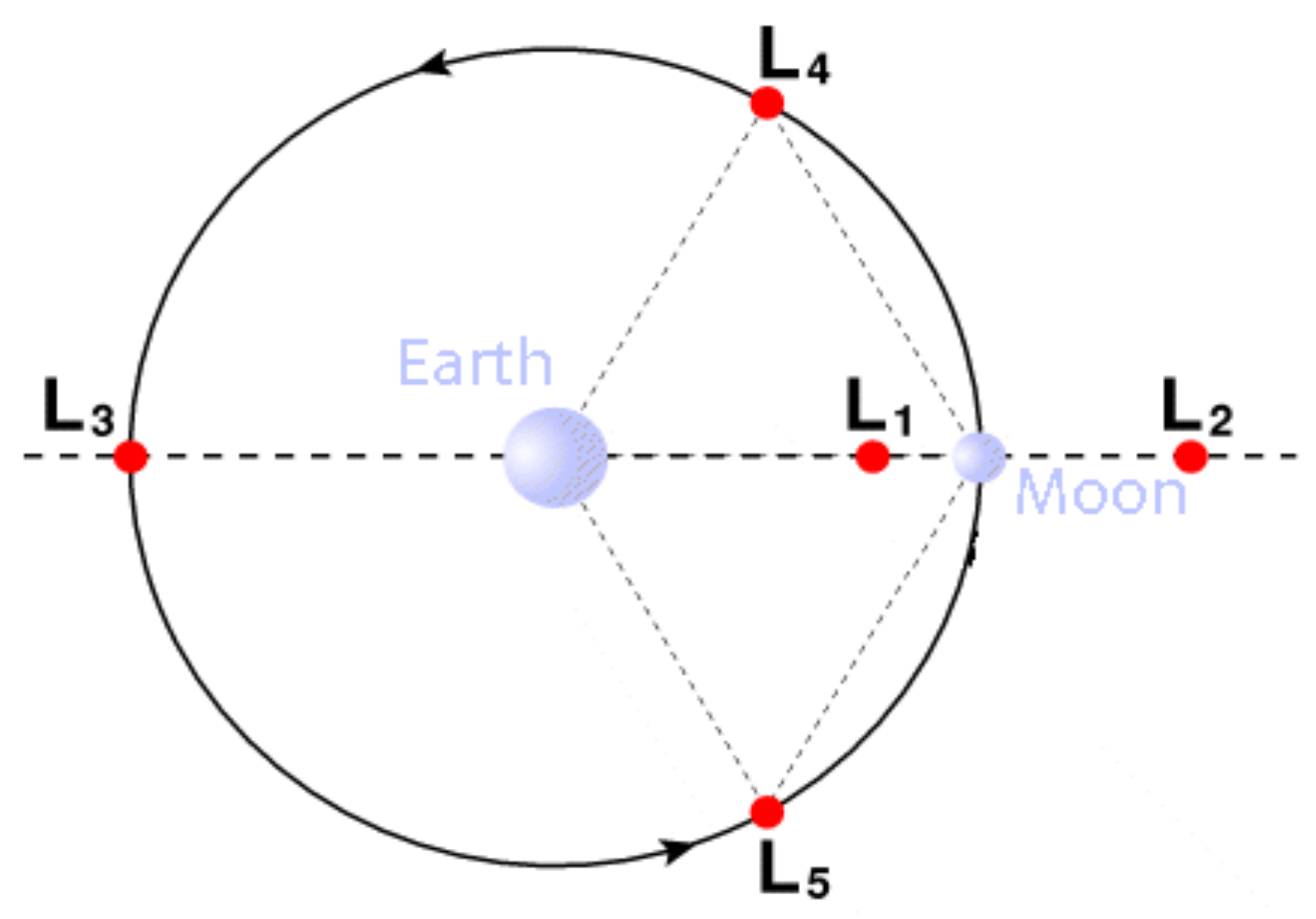
$$k_2 \equiv \kappa_2 \frac{G\hbar}{c^3} = \kappa_2 (l_P)^2,$$

$$l_P = \sqrt{\hbar G/c^3} \approx 1.616 \times 10^{-35}\mathrm{m}$$

# Classical Lagrangian points

$L_1$, $L_2$, and $L_3$: unstable

$L_4$ and $L_5$: stable



# Quantum corrections on Lagrangian points (1)

- Quantum corrections on Newtonian non-collinear Lagrangian points:

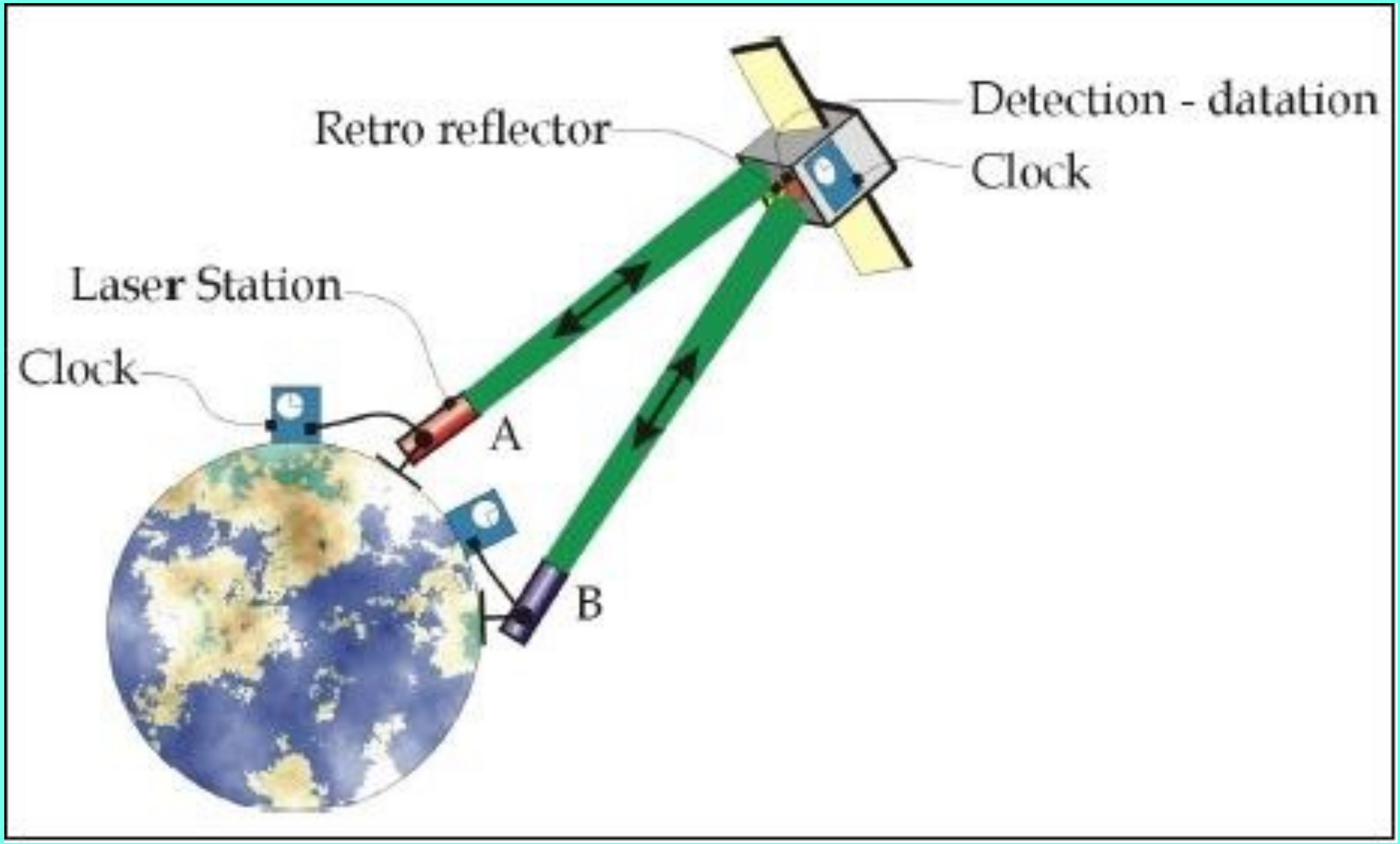
| Quantum corrections on Newtonian non-collinear Lagrangian points | | | |
|---|---|---|---|
| $L_i$ | One-particle reducible | Scattering | Bound-states |
| $L_4$ | $r_Q - r_{cl} = -2.96$ mm | $r_Q - r_{cl} = 8.87$ mm | $r_Q - r_{cl} = -1.48$ mm |
| | $x_Q - x_{cl} = -2.92$ mm<br>$y_Q - y_{cl} = -1.73$ mm | $x_Q - x_{cl} = 8.76$ mm<br>$y_Q - y_{cl} = 5.18$ mm | $x_Q - x_{cl} = -1.46$ mm<br>$y_Q - y_{cl} = -0.864$ mm |
| $L_5$ | $r_Q - r_{cl} = -2.96$ mm | $r_Q - r_{cl} = 8.87$ mm | $r_Q - r_{cl} = -1.48$ mm |
| | $x_Q - x_{cl} = -2.92$ mm<br>$y_Q - y_{cl} = 1.73$ mm | $x_Q - x_{cl} = 8.76$ mm<br>$y_Q - y_{cl} = -5.18$ mm | $x_Q - x_{cl} = -1.46$ mm<br>$y_Q - y_{cl} = 0.864$ mm |

# Quantum corrections on Lagrangian points (2)

- Collinear Lagrangian points:

| Quantum corrections on Newtonian collinear Lagrangian points | | | |
|---|---|---|---|
| $L_i$ | One-particle reducible | Scattering | Bound-states |
| $L_1$ | $r_Q - r_{cl} = -1.23$ mm<br>$x_Q - x_{cl} = -1.23$ mm | $r_Q - r_{cl} = 3.70$ mm<br>$x_Q - x_{cl} = 3.70$ mm | $r_Q - r_{cl} = -0.617$ mm<br>$x_Q - x_{cl} = -0.617$ mm |
| $L_2$ | $r_Q - r_{cl} = -0.783$ mm<br>$x_Q - x_{cl} = -0.783$ mm | $r_Q - r_{cl} = 2.35$ mm<br>$x_Q - x_{cl} = 2.35$ mm | $r_Q - r_{cl} = -0.392$ mm<br>$x_Q - x_{cl} = -0.392$ mm |
| $L_3$ | $r_Q - r_{cl} = -2.96$ mm<br>$x_Q - x_{cl} = 2.96$ mm | $r_Q - r_{cl} = 8.89$ mm<br>$x_Q - x_{cl} = -8.89$ mm | $r_Q - r_{cl} = -1.48$ mm<br>$x_Q - x_{cl} = 1.48$ mm |

# Laser ranging technique




- Very short pulse of light is fired towards satellites equipped with cube corner retro-reflectors.
- The round trip time of flight is measured.

# Boosting the metric (1)

- In 1971 Aichelburg and Sexl developed a method to describe the gravitational field associated to a massless point particle moving at the speed of light.

- Boosting procedure to Schwarzschild-de Sitter metric ($\Lambda = \frac{3}{a^2}$)

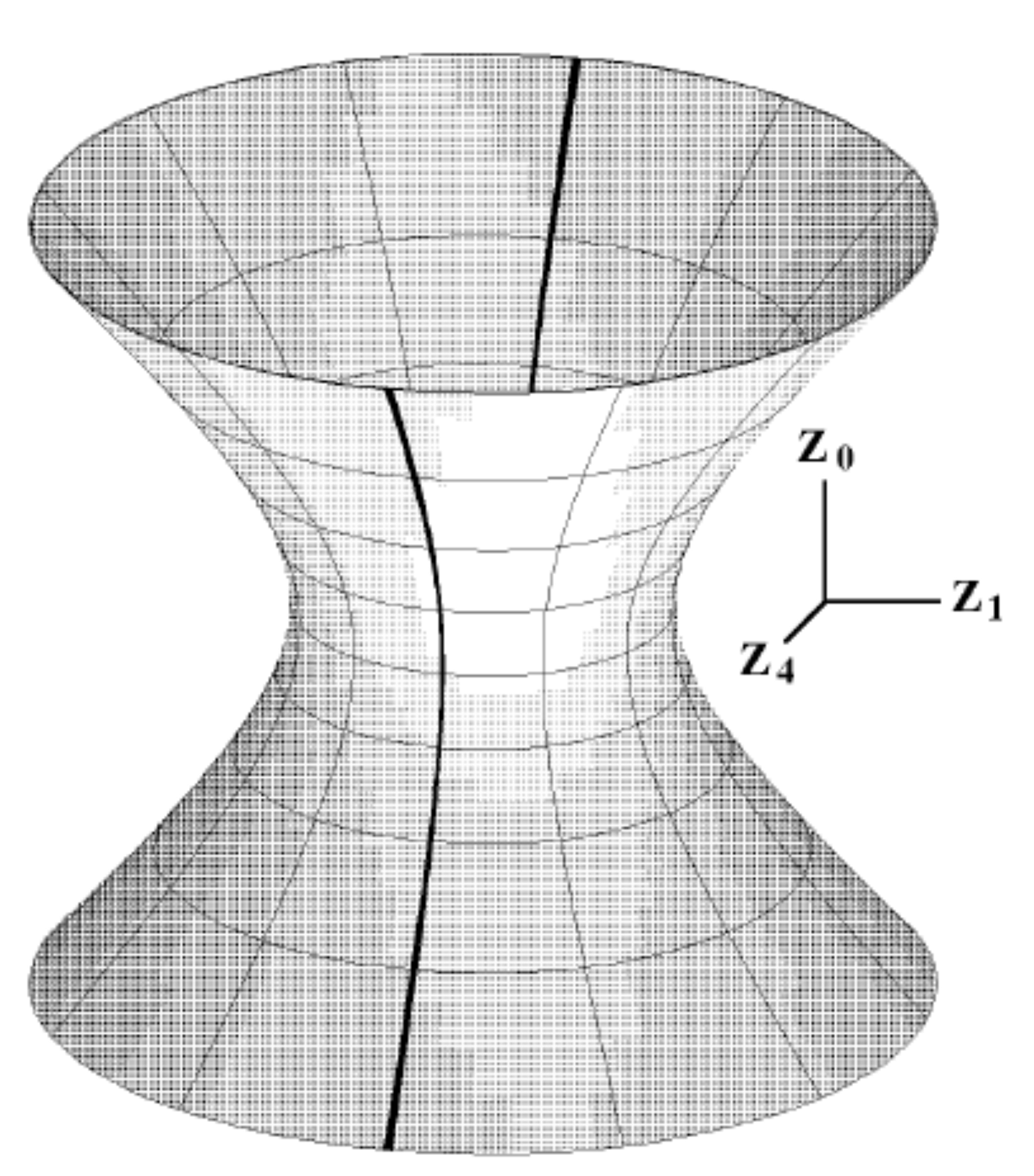
$$\mathrm{d}s^2 = -\left(1 - \frac{2m}{r} - \frac{r^2}{a^2}\right)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{\left(1 - \frac{2m}{r} - \frac{r^2}{a^2}\right)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)$$

(*G*=*c*=*1*)

- A de Sitter spacetime in four dimensions can be considered as a four-dimensional hyperboloid embedded in a five-dimensional Minkowski spacetime

# Boosting the metric (2)

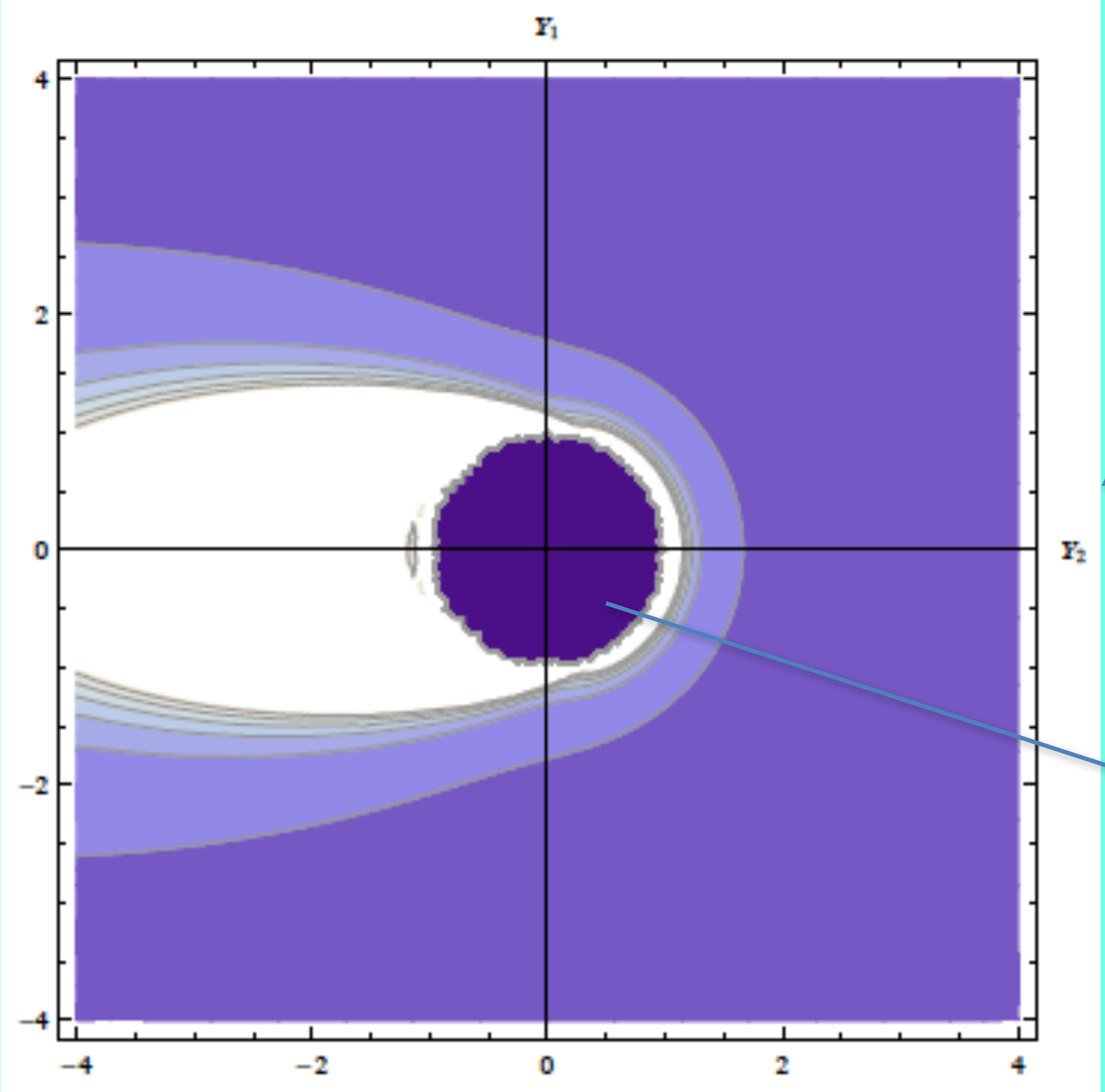
(two dimensions suppressed)



# The Kretschmann invariant

The Kretschmann invariant is not defined unless

$$Y_1^2 + Y_2^2 + Y_3^2 + Y_4^2 > a^2$$




Contour plot in the $Y_1$-$Y_2$ plane

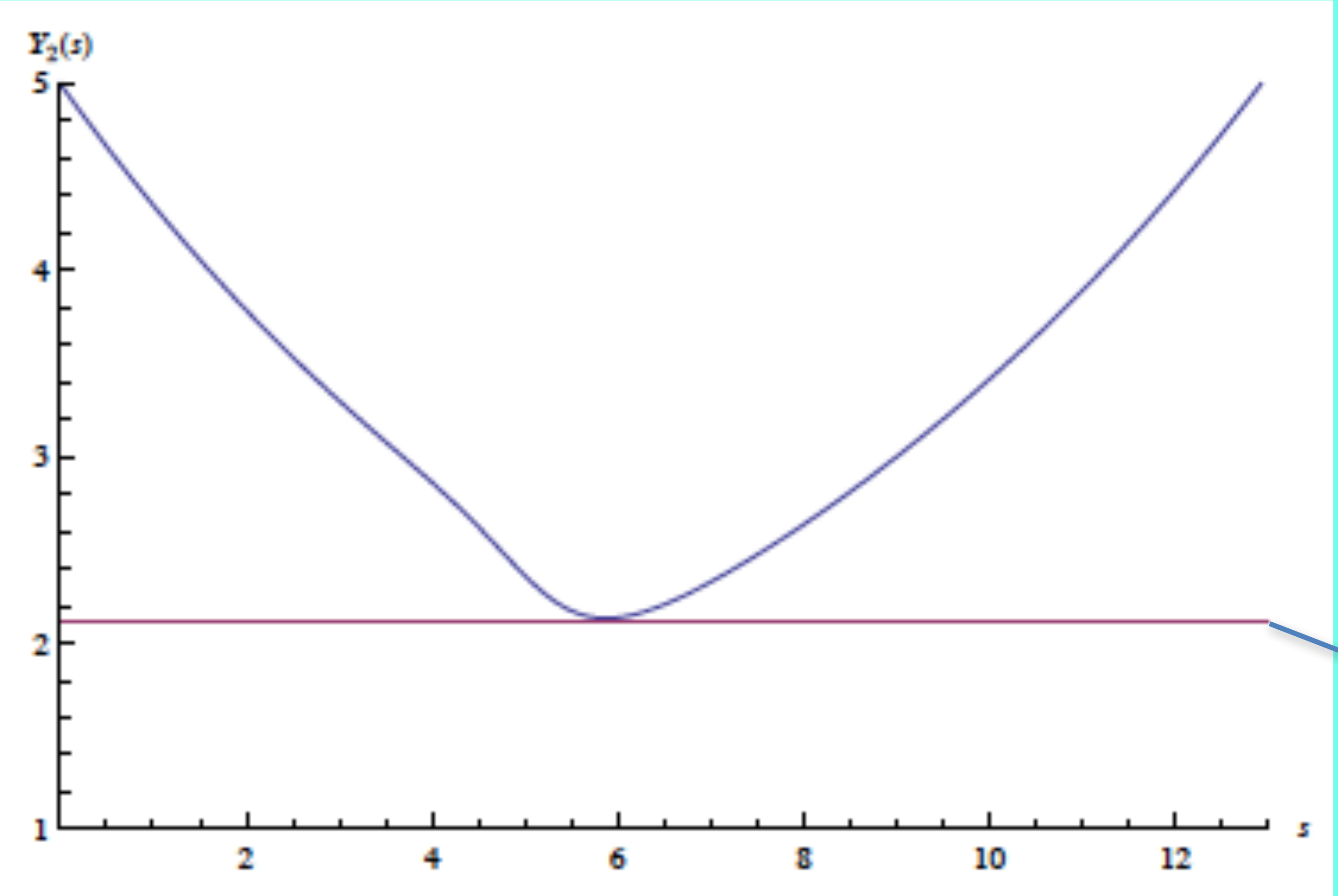
Circle of radius *a* where the Kretschmann invariant is not defined

# The “boosted horizon”

Geodesic equation

$$\ddot{Y}^\mu(s) + \Gamma^\mu_{\nu\lambda}\dot{Y}^\nu(s)\dot{Y}^\lambda(s) = 0$$

“Boosted horizon”: a barrier, surrounding the 3-sphere of radius *a*, where all geodesics, despite maintaining their completeness condition, are pushed away $\longrightarrow$ antigravity effect




Initial Conditions:

$Y_1(0)=Y_3(0)=Y_4(0)=0,\ Y_2(0)=5;$

$Y'_1(0)=Y'_3(0)=Y'_4(0)=0,\ Y'_2(0)=-0.7;$

$a=1,\ m=1,\ v=0.9.$

location of the “boosted horizon”

# Conclusions (1)

- We have derived the following GR and quantum corrections on Lagrangian points

| $L_i$ | General Relativity-Newton | Quantum-General Relativity | Quantum-Newton |
|---|---|---|---|
| $L_1$ | 0.19 mm | -0.62 mm | -0.43 mm |
| $L_2$ | -0.32 mm | -0.39 mm | -0.71 mm |
| $L_3$ | -0.04 mm | -1.48 mm | -1.52 mm |
| $L_4$ | (2.73 mm, -1.59 mm) | (-1.46 mm, -0.86 mm) | (1.27 mm, -2.45 mm) |
| $L_5$ | (2.73 mm, -1.59 mm) | (-1.46 mm, -0.86 mm) | (1.27 mm, -2.45 mm) |

# Conclusions (2)

- Theoretical predictions presented here are testable in light of modern advances in lunar laser-ranging techniques, but several perturbations, of gravitational and non-gravitational nature, may (slightly) modify such predictions.

$\downarrow$

NEWREFLECTIONS experiment

# Conclusions (3)

- I have described the procedure of boosting a metric and I have explained how this can be applied to the Schwarzschild-de Sitter metric.
- I have brought the boosted Schwarzschild-de Sitter metric to its usual four-dimensional form and then I have applied numerical calculation to this form of the metric.

- Main results

The singularity 3-sphere where the Kretschmann invariant is not defined

The "boosted horizon" (antigravity effects)

# Conclusions (4)

- References:


1. E. Battista, *"Extreme Regimes in Quantum Gravity"*, PhD thesis, arXiv:1606.04259.

2. E. Battista, *"Extreme Regimes in Quantum Gravity"*, Nova Science Pulishsers, New York, ISBN 978-1-53612-451-4.